**UNIT-I DISCRETE FOURIER TRANSFORM**

Review of signals and systems, concept of frequency in discrete-time signals, summary of analysis & synthesis equations for FT & DTFT, frequency domain sampling, Discrete Fourier transform (DFT) - deriving DFT from DTFT, properties of DFT - periodicity, symmetry, circular convolution. Linear filtering using DFT. Filtering long data sequences - overlap save and overlap add method. Fast computation of DFT - Radix-2 Decimation-in-time (DIT) Fast Fourier transform (FFT), Decimation-in-frequency (DIF) Fast Fourier transform (FFT). Linear filtering using FFT.

deriving DFT from DTFT, properties of DFT - periodicity, symmetry, circular convolution. Linear filtering using DFT. Filtering long data sequences - overlap save and overlap add method. Fast computation of DFT - Radix-2 Decimation-in-time (DIT) Fast Fourier transform (FFT), Decimation-in-frequency (DIF) Fast Fourier transform (FFT). Linear filtering using FFT.

**Introduction:**

**Signal:** Signal is defined as any physical quantity that varies with time, space or any other independent variable.

**System:**

A system is defined as physical device that generates a response or an output signal, for a given input signal.

X(t) y(t)

System

 input output

where,

y(t) ----> operation on x(t)

y(t) = T[x(t)]

**Types of Signal:**

There are two types of signal, that are based on time because the signal basically varies with respect to time, they are,

* Continuous time signal
* Discrete time signal

**Continuous time signals (CT):**

CT Signals are continuously varied in accordance with the time.

**Example: Sine wave, cosine wave etc.**



 **Cosine wave**

**Discrete time signals (DT):**

DT signal is defined as at discrete instants of time are known as discrete-time signals. The discrete-time signals are continuous in amplitude and discrete in time. They are denoted by x(n).

**Example:**

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**Basic Elements of DSP:**

|  |
| --- |
| **1. Explain briefly Basic Elements of Digital Signal Processing.**  |

Digital signal processing system contains following elements are,

* Analog signal
* Sample and Hold circuit
* ADC (Analog to Digital Converter)
* Filter
* DAC (Digital to Analog Converter)

Sample and Hold Circuit

ADC

Filter

DAC

* Analog signal is nothing but real time signal. It may be speech, video etc.
* Analog signal (continuous –time signal) is given to sample and hold circuit.
* After completing the sampling process, the signal is given to ADC block.
* ADC converts Analog to Digital signal. Then it is given to filter block. This filter removes unwanted noise from the signal. Then the output is processed through the channel.
* In the receiver side (Rx), digital signal is again converted into analog signal. Because analog signal is the real time signal and we can understand only analog signal.

**Concepts of frequency in Analog and Digital Signals: (Continuous-time and discrete time signals**

**2. Discuss the concepts of frequency in Analog and Digital Signals and its Properties.**

A simple harmonic oscillation is mathematically described by the following continuous-time sinusoidal signal.

Where,

A 🡪Amplitude of the sinusoid.

Ω 🡪Frequency in radian/Seconds; Ω = 2πF

θ 🡪Phase in radians.

Xa(t) 🡪Analog signal representation.

In terms of F, eqn(1) can be written as,



Where,

T 🡪Time period in sec.

t 🡪 frequency in Hz.

X(t) 🡪 signal represent after the time period T. frequency ‘f’ satisfies the relationship.

We can prove that,

Add equation (5) & (6),

 = = 2

So, = [ ]

Result is

**(ii) Discrete- time signals:**

A discrete-time sinusoidal signal may be expressed as,

 ----------> (1)

Where,

n 🡪 An integer variable.

A 🡪 Amplitude of the sinusoid.

 -------> Frequency in radians per sample

 --------> Phase in radians.

If instead of , we use the frequency variable ‘f’.

 ----------------------------------------------> (2)

Equation (1) becomes,

 --------> (3)

The frequency ‘f’ has dimensions of cycle per sample.

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In contrast to continuous-time sinusoids, the discrete-time sinusoids are characterized by the following properties.

**Properties:**

* A discrete-time is periodic only if its frequency is a rational number.
* Discrete-time sinusoid whose frequencies are separated by an integer multiple of 2π are identical.
* The highest rate of oscillation in a discrete-time sinusoid is attained. When ( (or) ( or equivalently (

**SAMPLING THEOREM:**

**3. Derive an expression for Sampling theorem and its Reconstruction of *x(t)*. [Nov/Dec-2010][May/June-12][May/june-2015]**

Any arbitrary band-limited continuous time signal can be represented in its samples and recovered from its samples taken at equal intervals at the sampling rate of fs≥2f samples/ second. This sampling theorem and reconstruction formula require infinite number of samples. But practically it is not possible to take infinite number of samples.

When the analog frequency is band limited to the range given by the sampling theorem. The digital frequency response is also changed in the same manner.

If fs<2f then aliasing problem will occur.

**Aliasing effect:**

Aliasing is a problem due to interference of information between two band of frequencies. The output due to aliasing is known as an aliased representation of the original signal. This effect should be avoided.

**How to avoid aliasing effect?**

According to sampling theorem, sampling the continuous time waveform at a high rate is the only one way to avoid aliasing effect.

**Nyquist rate:**

The sampling rate is generally referred as Nyquist rate.

**Proof of sampling theorem:**

Consider *x(t)* as input continuous signal. It has finite energy and finite duration. This *x(t)* is band limited signal.

Ts=1/fs = Sampling period

fs= Sampling frequency

Impulse function is given as

 

After sampling input x(t) is represented as 

Fourier transform of equation is given as







**Find FT of *x(t)***

If we take the Fourier transform of continuous signal *x(t)*, we will get

If we write above equation for discrete signal put t=nTS



Rearrange equation (5)





If frequency lies between –f to +f

Put eqn (7) in (9)



**Reconstruction of *x(t)*:**

x(t) can be reconstructed from equation (10) and put fS=2f



Take inverse Fourier transform of above equation





(Order of summation and integration is interchanged)

In above equation put formula, which is the sin c function



Now,



The above equation is also known as interpolation formula and it expand as,

 

**Challenge: 1**

**Consider the analog signal .**

**(a) Determine the minimum sampling rate required to avoid aliasing.**

**(b) Find the nyquist rate and Nyquist interval.**

**(c) Find the folding frequency.**

**Given:** Analog signal .

 General equation .

Compare with given equation, A= 3;

1. **Determine the minimum sampling rate required to avoid aliasing.**

This is the maximum frequency fmax (or)

Minimum sampling rate

1. **Find the Nyquist rate and Nyquist interval.**

**Nyquist rate =**

**Nyquist Interval =**

1. **Find the folding frequency.**

Half of the sampling frequency =

**Discrete – Time Signals:**

* Representation of Discrete – Time Signals:
* Elementary of Discrete – Time Signals:
* Classification of Discrete – Time Signals:
* Operation of Signals:

**Representation of Discrete – Time Signals:**

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| --- |
| **4. Explain the different representation of Discrete-time signals. [Nov/Dec-13]** |

There are different types of representation for discrete-time signals. They are

* Graphical representation
* Functional representation
* Tabular representation
* Sequence representation

**Graphical representation:**

Let us consider a signal x(n) with values and . This discrete-time signal can be represented graphically as shown in below.

 (n) 2

 1.5

 1

 0.5

 -1 0 1 2 3 n

**Functional Representation:**

The discrete-time signal can be represented using functional representation is below.



**Tabular Representation:**

The discrete-time signal can also be represented as,

n -1 0 1 2 3

x(n) 1 2 2 0.5 1.5

**Sequence Representation:**

A finite duration sequence with time origin (n = 0) indicated by the symbol is represented as



An infinite duration sequence can be represented as,



A finite duration sequence that satisfies the condition for n<0 can be represented as,



**Elementary of Discrete – Time Signals:**

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| --- |
| 5. Explain the different representation of Discrete-time signals. |

There are different types of elementary of discrete-time signals are,

* Unit step sequence
* Unit ramp sequence
* Unit-sample sequence
* Exponential sequence
* Sinusoidal sequence
* Complex exponential sequence.

**Unit step sequence:**

The unit step sequence is defined as

The graphical representation of u(n) is shown in figure.

**Unit ramp sequence:**

The unit ramp sequence is defined as

The graphical representation of r(n) is shown in figure



**Unit-sample sequence (unit impulse sequence):**

The unit-sample sequence is defined as,

The graphical representation of is shown in figure.

The unit impulse unction has the following properties.

-----------------> (1)

-----------------------> (2)

---------> (3)

**Exponential sequence:**

The exponential signal is a sequence of the form

Different types of discrete-time exponential signals.

* When the value of a > 1, the sequence grows exponentially and
* When the values is 0 < a < 1, the sequence decay exponentially.
* When a < 0, the discrete-time exponential signal takes alternating signs.



**Sinusoidal signal:**

The discrete-time sinusoidal signal is given by,

---------> (1)

Where, - is the frequency (in radians per sample) and  is the phase (in radians).

Using euler’s identity, we can write

-------> (2)

Since , the energy of the signal is infinite and the average power of the signal is 1.

**Complex Exponential signal:**

The discrete-time complex exponential signal is given by

------ 🡪(1)



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**Classification of Discrete-time signals:**

There are different types of classification of discrete-time signals are,

* Energy signals and power signals.
* Periodic and Aperiodic signals
* Symmetric(even) and ant-symmetric(odd) signals.
* Causal and non-causal signals.

**Energy signals and power signals:**

**Energy signal:**

A discrete-time signal x(n) the energy ‘E’ is defined as,

**Power signal:**

The average power of a discrete-time signal x(n) is defined as,

**Note:**

* A signal is energy signal, if an only if the total energy of the signal is finite. For an energy signal P=0.
* A signal is power signal, if the average power of the signal is finite. For power signal E=.
* The signals that do not satisfy above properties are neither energy nor power signals.

**Challenge 1:**

**Determine the values of power and energy of the following signals. Find whether signals are power, energy or neither energy nor power signals.**

1. **[May/June-2016]**

**Solution:**

**Given:**

1. signal

To find Energy signal of x(n)

 

To find Power of x(n) :

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**Result:** The energy is finite and power is zero. Therefore, the signal is an energy signal.

**Solution:**

**Given:** signal (ii)

To find Energy of x(n):



To find power of x(n):



**Result**: The energy is infinite and power is finite. Therefore, the signal is power signal.

**H.W: Find whether the signals are power, energy or neither energy nor power signals.**

**(i)  Ans: Power signal**

**(ii)  Ans: Energy signal**

1. **(iii) Ans : Power signal**
2. **Ans : Power nor energy signal**

**Periodic and Aperiodic Signals:**

**Challenge 1:**

**Determine whether the signal is periodic or not. If the signal is periodic, find the fundamental period.**

1. ** (ii)  (iii)  (iv)** 

**Solution:**

**Given signal: .**

The fundamental frequency is multiple of π. Therefore, the signal is periodic.

****Therefore the fundamental period is 3.

**(ii) **

**Solution:**

**Given Signal: ;** , which is not a multiple of . Therefore, the signal is Aperiodic.

**H.W.Determine the fundamental period of the following signals, if they are priodic.**

**(i) Ans:Periodic with N=8**

**(ii) Ans:Aperiodic**

**(iii) Ans:Aperiodic**

**(iii) Ans : Periodic with N=3**

**(iv)**  **Ans : Periodic with N=24**

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**Symmetric (even) and antisymmetric(odd) signals:**

**Symmetric:**

A discrete-time signal x(n) is said to be a symmetric (even) signal, if it satisfies the condition.

**------> (1)**

Example: cos

**Antisymmetric:**

The signal is said to be an odd signal if it satisfies the condition.

**--------> (2)**

**Example:** sin

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**Causal and Non-causal Signals:**

A signal is said to be causal, if its value is zero for n < 0. Otherwise the signal is non-causal.

**Basic operation on Signals:**

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| **6. Explain the basic operation on Signals.** |

Signal processing is a group of basic operations applied to an input signal resulting in another signal as the output.

* The mathematical transformation from one signal to another is represented as



The basic sets of operations are,

1.Shifting

2. Time reversal

3. Time scaling

4. Scalar multiplication

5 Signal multiplier

6. Signal addition

**Shifting:**

* The shift operation takes the input sequence and shifts the values by an integer increment of the independent variable.
* The Shifting may delay or advance the sequences in time. Mathematically this can be represented as

y(n) = x(n - k)

Where x(n) is the input and y(n) is the output.

* If k is positive. the shifting delays the sequence.
* If k is negative. the shifting advances the sequence.

**Example:** The signal x(n - 3) is obtained by shifting x(n) right by 3 units of time. The result is shown in Figure. On the other hand, the signal x(n+2) is obtained by shifting x(n) left by two units of time.

 **Time reversal:**

* The time reversal of sequence x(n) can be obtained by folding the sequence about n = 0.It is denoted as x(-n). For the signal x(n) shown in figure, the x(-n) is given in figure.

**Example:** The signal x(-n + 2) is x(-n) delayed by two units of time and x(-n- 2) is x(-n) advanced by two units of time. The graphical representation of x(-n - 2) and x(-n + 2) are shown in figure.



**Time Scaling:**

This is accomplished by replacing ‘n’ by λn in the sequence x(n). Let x(n) is a sequence shown in figure.

If λ = 2 we get a new sequence

**y(n) = x(2n).**

We can plot the sequence y(n) by substituting different values for n.

For 

Similarly, 



**Scalar Multiplication:**

A scalar multiplier is shown in below. Here, the signal x(n) is multiplied by a scale factor a.



For example if then the signal

**Signal Multiplier:**

The multiplication of two signal sequence to form another sequence.



For example, if  

Then, 

**Addition Operation:**

Two signals can be added by using an adder shown in below.

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For example, if



Then, 

**Classification of Discrete-time systems:**

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| --- |
| 7. Discuss the various Classifications of Discrete-time systems. [May/June-2013][Nov/Dec-13][April/May-14][Nov/Dec-14] |

Discrete-time systems are classified according to their general properties and characteristics. They are,

* Static and Dynamic systems
* Causal and Non-causal systems
* Linear and Non-linear systems
* Time variant and Time-Invariant systems
* Stable and Unstable systems.

**Static and Dynamic systems**

Static systems:

* A Discrete-time system is called Static or Memory less if its output at any instant ‘n’ depends on the input samples at the same time, but not on past or future samples of the input. The output at any instant depends on the input at that instant.

Example:



Dynamic systems:

* A Discrete-time system is said to be dynamic or to have memory, if the output of y(n) depends on past or future samples of the input. The output depends on past values of input. It requires memory.

Example:

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**Causal and Non-causal Systems:**

Causal system:

A system is said to be causal if the output of the system at any time n [i.e y(n)] depends only on present and past inputs [i.e., x(n), x(n-1, x(n-2)…)]but does not depend on future inputs [i.e., x(n+1, x(n+2)…)]. In mathematical terms, the output of a casual system satisfies an equation of the form

where F[.] is some arbitrary function.

Example:

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Non-Causal systems:

If the output of a system depends on future inputs, the system is said to be non-causal or anticipatory.

Example:

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**Linear system and Non-linear systems:**

Linear system:

A linear system is the one that satisfies the superposition principle. Superposition principle states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of responses (output) of the system to each of the individual input signals.

A system is linear if and only if

where x1(n) and x2(n) are arbitrary input sequence and a1 and a2 are any arbitrary constants .

Example:

*y(n) = n x(n)*

Non-linear systems:

A system does not satisfy the superposition principle is called non- linear system.

Example:

*y(n)=Ax(n) + B*

**Time Variant and Time-Invariant systems:**

Time invariant (Shift Invariant)system:

A relaxed system is time invariant or shift invariant if and only if

Implies that

For every input signal x(n) and every time shift k.

In other words, A system is said to be time-invariant or shift invariant if the characteristics of the system do not change with time.

i.e 

Example:

Differntiator

Time variant (Shift variant)system:

A system is said to be time-variant or shift variant if the characteristics of the system changes with time.

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**Stable and Unstable systems:**

Stable system:

An arbitrary relaxed system is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces a bounded output.



Unstable System:

If, for some bounded input sequence x(n), the output is unbounded(infinite), the system is classified as unstable.

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**Challenge 1: find whether the following systems are static or dynamic.**

**(i) (ii)**

**Solution:**

Given: Output system ****

The output y(n) depends on the past input. Therefore the system is dynamic.

(ii) ****

**Solution:**

Given: output system ****

The output y(n) depends on the present input only. Therefore the system is static.

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**H.W: check whether the following systems are static or dynamic.**

(i)  Ans: Dynamic

(ii)  Ans: Static

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**Challenge 2: Test whether the following systems are causal or non-causal.**

**(i) (ii) **

**Solution:**

Given: output of the system 

For n=-1; 

For n=0;  

For n=1; 

For all values of ‘n’, the output depends on present and past inputs. Therefore, the system is causal.

**(ii) **

**Solution:**

Given: output of the system ****

For n= -1; 

For n=0; 

For n= 1; 

For all values of ‘n’, the output depends on future inputs. So, the system is Non-causal.

H.W: **Test whether the following systems are causal or non-causal.**

(i)  Ans: Causal

(ii) Ans: Causal.

**Challenge 3: Determine if the system described by the following input-output equations is linear or non-linear.[May/June-2016]**

**(i) (ii) **

**Solution:**

Given: output of the system 

For two input sequences the corresponding outputs are,

--------------------------------------------------------> (1)

------------------------------------------------------> (2)

The output due to weighted sum of input is

-----> (3)

The linear combination of the two output is,

------------------------> (4)

equation (3) and (4) are not equal. So, the system is non-Linear.

**(ii)** 

Solution:

Given: output of the system 

For two input sequences the corresponding outputs are,

---------------------------------------------------------------------> (1)

-------------------------------------------------------------------> (2)

The output due to weighted sum of input is

------------------------------------> (3)

The linear combination of the two output is,

------------------------------------------------> (4)

equation (3) and (4) are equal. So, the system is Linear.

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**H.W: check whether the system is linear or not.**

**(i)  Ans: Non-linear**

**(ii)  Ans: Non-linear**

**(iii)  Ans: Non-linear**

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**Challenge 4: Determine if the following systems are time-invariant or time variant.**

**(i) (ii) **

**Solution:**

Given: output of the system 

If the input is delayed by ‘k’ units in time, we have

------------------------------------------------------------> (1)

If the output is delayed by ‘k’ units in time, then (n-> n-k)

----------------------------------------------------------> (2)

Here, 

Therefore, the system is time-invariant.

**(ii) **

Solution:

Given: output of the system 

If the input is delayed by ‘k’ units in time, we have

---------------------------------------------------------------------------> (1)

If the output is delayed by ‘k’ units in time, then (n-> n-k)

-------------------------------------------------------------------------> (2)

Here, 

Therefore, the system is time-variant.

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**H.W: Determine if the following systems are time invariant.**

**(i)  Ans: Time-variant**

**(ii)  Ans: Time-Variant**

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**Challenge 5: Test the stability of the system whose impulse response **

**Solution:**



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Hence the system is stable.

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**Challenge 6: Test if the following systems are stable or not.**

**(i) (ii) **

**Solution:**

Given: output of the system 

If the input x(n) is bounded,  then



That is hence the system is stable.

**(ii)**

**Solution:**

Given: output of the system 

If the input x(n) is bounded,  then



The output increases with increasing n. hence the bounded input produce unbounded output. Hence the system is unstable.

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**H.W: Test if the following systems are stable or not.**

**(i)  Ans: Stable**

**(ii)  Ans: Stable**

**(iii)  Ans: Stable**

**(iv) Ans: Stable**

**(v) Ans: Stable**

**(vi) Ans: Stable**

**(vii) ** **Ans: Stable**

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**DFT (Discrete-Fourier Transform)**

The sequence X(k ) is called the N-point DFT of x(n). These coefficients are related to x(n) as follows:

The N-point IDFT of the sequence X(k) is

**Challenge 1.**

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**Challenge 2:Determine the 8-point DFT of the sequence x(n)={1,1,1,1,1,1,0,0}.[Nov/Dec -2010][April/May-2011]**

**Solution:**

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For N=8



For k=0



 X(0) = x(0)+x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)

= 1+1+1+1+1+1+0+0

= 6

For k=1



X(1) = x(0) +x(1) e-jπ/4+x(2) e-jπ/2+x(3) e-j3π/4+x(4) e-jπ+x(5) e-j5π/4+x(6) e-j3π/2+x(7) e-j7π/4

 = 1+0.707-j0.707-j-0.707-j0.707-1-0.707+j0.707 => -0.707-j1.707

For k=2



X(2) = x(0)+x(1) e-jπ/2+x(2) e-jπ+x(3) e-j3π/2+x(4) e-j2π+x(5) e-j5π/2+x(6) e-j3π+x(7) e-j7π/2

 = 1-j-1+j+1-j

 = 1-j

For k=3



X(3) = x(0)+x(1) e-j3π/4+x(2) e-j3π/2+x(3) e-j9π/4+x(4) e-j3π+x(5) e-j15π/4+x(6) e-j9π/4 +x(7) e-j21π/4

 = 1-0.707-j0.707+j+0.707-j0.707-1+0.707+j0.707

 = 0.707 + j0.293

For k=4



X(4) = x(0)+x(1) e-jπ +x(2) e-jπ2 +x(3) e-jπ3 +x(4) e-jπ4 +x(5) e-jπ5 +x(6) e-jπ6 +x(7) e-jπ7

 = 1-1+1-1+1-1 = 0

For k=5



X (5) = x(0)+x(1) e-j5π/4+x(2) e-j5π/2+x(3) e-j5πn/4+x(4) e-j5π+x(5) e-j25π/4+x(6) e-j15π/2 +x(7) e-j35π/4

 = 1-0.707+j0.707-j+0.707+j0.707-1+0.707-j0.707

 = 0.707-j0.293

For k=6



X(6) = x(0)+x(1) e-j3π/2+x(2) e-j3π+x(3) e-j9π/2+x(4) e-j6π+x(5) e-j15π+x(6) e-j9π+x(7) e-j21π/2

 = 1+j-1-j+1+j => = 1+j

For k=7



X(7) = x(0) +x(1) e-j7π/4+x(2) e-j7π/2+x(3) e-j21π/4+x(4) e-j7π+x(5) e-j35π/4+x(6) e-j21π/2 +x(7) e-j49π/4

 = 1+0.707+j0.707+j-0.707+j0.707-1-0.707-j0.707

 = -0.707+j1.707

X(K)={6,0.707-j1.707,1-j,0.707+j0.293,0,0.707-j0.293,1+j,-0.707+j1.707}

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**H.W: 1. Find the DFT of the Sequence  ; Ans: **

**2. Determine the 4-point DFT of the sequence; Ans: **

**Challenge 3: Find IDFT of the sequence X (K) = (5,0,1-j,0,1,0,1+j,0)**

**Solution:**

We have

x(n)=

For N=8

x(n)=

For n=0; x(0)== [5+0+1-j+0+1+j+0]=1

For n=1; x(1)==[5+-(1-j)(j)+1(-1)+(1+j)(-j)]= 6/8= 0.75

For n=2; x(2)==[5+(1-j)(-1)+1(1)+(1+j)(-1)]=4/8 =0.5

For n=3; x(3)== [5+(1-j)(-j)+1(-1)+ (1+j)(j)]= 2/8 = 0.25

For n=4; x(4)==[5+(1-j)(1)+1(1)+(1+j)(1)]=8/8=1

For n=5; x(5)= =[5+(1-j)(j)+(1)(-1)+(1+j)(-j)]=6/8=0.75

For n=6; x(6)==[5+(1-j)(-1)+1(1)+(1+j)(-j)]=4/8=0.5

For n=7;x(7)==[5+(1-j)(-j)+1(1)+(1+j)(j)]=2/8=0.25

x(n) = {1,0.75,0.5,0.25,1,0.75,0.5,0.25}

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\* **Challenge 4: Find IDFT of the sequence X (K) = {1,0,1,0}**

**Solution:**

We have



For N=4



For n=0







For n=1



For n=2



For n=3





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**H.W: 1. Find the IDFT of the sequence  with N=4.**

**Ans:** *x(n)=*[]

**2. Find the 4-point IDFT of the sequence ; Ans: **

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**Challenge 5: Perform the circular convolution of the following sequences**

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**Solution:**

Given: the circular convolution of the following sequences

****

**To find DFT of *x1(k):***

****

**To find DFT of *x2(k)*:**

****

****

**To find IDFT of *X3(n)*:**

****

**\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\***

**Properties of DFT:**

|  |
| --- |
| **1. State and prove any four properties of DFT. May/June-2012] [Nov/Dec-2012] [May/June-2013][Nov/Dec-2013] [Nov/Dec-2014]** |

1. Periodicity property

2. Linearity property

3. Symmetry property

4. Multiplication of two DFT and convolution

5. Time reversal sequence

6. Parseval’s theorem

**Periodicity property**

If X(k) is N-point DFT of a finite duration sequence x(n)

Then

**Linearity**

If two finite duration sequences x1(n) and x2(n) are linearly combined as

Then the DFT of x3(n) is

**Proof:**

In summary if,

Then

**Time Reversal of the sequence**

**Proof:**

Changing the index from n to m = N-n, we get

**Complex conjugate property**

If then

Proof:

**Multiplication of Two sequences**

If and

 then

**Parseval’s Theorem:**



**FFT-Fast Fourier Transform**

* DFT is given by



* The direct computation of the DFT requires 2N2 evaluations of trigonometric functions, 4N2 real multiplications 4N (N-1) real additions.

By using the twiddle factor,



Where, WN is the “Nth” root of unity.

**Properties of twiddle Factor:**

****

There are two types of Radix-2 FFT

1. Decimation in Time FFT
2. Decimation in Frequency FFT

**Decimation in Time FFT**

|  |
| --- |
| 2. Draw and explain the basic butterfly diagram or flow graph of DIT radix-2 FFT.[Nov/Dec-2009] |

1.In each computation two complex numbers” and “b” are considered.

2. The complex number “b” is multiplied by a phase factor “WNkn”

3. The product “ bWNkn” is added to complex number “a” to form new complex number “A”

4. The product “ bWNkn” is subtracted from comblex number “a” to form new complex number “B”.

The above basic computation can be expressed by a signal flow graph

The signal flow graph is also called butterfly diagram since it resembles a butterfly. In radix-2 FFT, Z/2 butterflies per stage are required to represent the computational process. The butterfly diagram used to compute the 8point DFT via radix-2 DIT FFT.

The sequence x (n) is arranged in bit reversed order and then decimated into two sample sequences.

x(0) x(2) x(1) x(3)

x(4) x(6) x(5) x(7)

Fist stage of flow graph for 8-point DFT via radix-2 DIT FFT

Second stage of flow graph for 8-point DFT via radix-2 DIT FFT

Third stage of flow graph for 8-point DFT via radix-2 DIT FFT

Combined stage for computation:

**Decimation in Frequency FFT**

|  |
| --- |
| 3. Draw and explain the basic butterfly diagram or flow graph of DIF radix-2 FFT.[Nov/Dec-2012] |

1**.** In each computation two complex numbers “a” and “b” are considered.

2. The sum of the two complex number a&b are considered.

3. The subtract complex number “b” from “a” to get the term “a-b” . The difference term “a-b” is multiplied with the phase factor or twiddle factor “WNk” to form a new complex number”B”.

The above basic computation can be expressed by a signal flow graph

The signal flow graph is also called butterfly diagram since it resembles a butterfly. In radix-2 FFT, Z/2 butterflies per stage are required to represent the computational process. The butterfly diagram used to compute the 8point DFT via radix-2 DIF FFT

Flow graph for Fist Stage of Computation:

Flow graph or butterfly diagram for second stage of computation:

Flow graph for Third Stage of computation:

**Challenge 6: Find the DFT of the sequence x[n] = {1, 2, 3, 4, 4, 3, 2, 1} using radix-2 decimation in time FFT algorithm. [Nov/Dec-2009] [April/May-2011][Nov/Dec-2012] [Nov/Dec-2012][May/June-2013] [Nov/Dec2013][April/May-2014][Nov/Dec-2014][May/June-2015] [May/June-2016].**

**Solution:**



**X(k)={2,0.5-j1.207,0,0.5-j0.207,0,0.5-j0.207,0,0.5-j1.207}**

**\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\***

**H.W: 1.Find the 8-point DFT of the sequence using DIT-FFT algorithm.**

**Ans: **

**2. Find the 8-point DFT of the sequence using DIT-FFT algorithm.**

**Ans: **

**3. Find the 8-point DFT of the sequence using DIT-FFT algorithm.**

**Ans: **

**4. Given, find *X(k)* using DIT-FFT algorithm.**

**Ans: **

**5. Compute the 8-point DFT of the sequence x (n) = (0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0) using the radix-2 DIT algorithm.**

**X(k)={2,0.5-j1.207,0,0.5-j0.207,0,0.5-j0.207,0,0.5-j1.207}**

**6. Find the 8-point DFT of the sequence  using DIT-FFT algorithm.**

 ******

**7. Given, find *X(k)* using DIT-FFT algorithm.**

**Ans: **

**Challenge 7: Compute 4-point DFT of a sequence using DIT algorithm.**

**Solution:**

Given: The DFT of the sequence ****

The twiddle factors are,



**

**

**Problem 8: Find the 8-point DFT of given sequence  using DIF-FFT algorithm. [May/June 2015] [Nov/Dec-2014]**



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**Challenge 8: Compute the DFT of the sequence ,where N=4 using DIF-FFT algorithm.**

**Solution:**

*Given:* Given: The DFT of the sequence **

The twiddle factors are,



**

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**\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\***

**H.W:1. Compute the FFT for the sequence  Ans: **

**2. Find the 8-point DFT of the given sequence using DIF, radix-2 FFT algorithm. Ans: **

**3. Find the 8-point DFT of the sequence using DIF-FFT algorithm.**

**Ans: **

**4.Find the IDFT of the sequence  using DIF algorithm. Ans: **

**5. Find the 8-point DFT of given sequence  using DIF-FFT algorithm. [May/June-2015] [Nov/Dec-2014]**

***X(k)=* {*12, 1-j2.414, 0, 1-j0.4142, 0, 1+j0.4142, 0, 1+j2.414}***

**6. Given, find *X(k)* using DIF-FFT algorithm.**

**Ans: **

**\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\***

**Filtering Methods based on DFT: Overlap-save method**

**Filtering of a long sequence**

 **Overlap-add method**

|  |
| --- |
| **4: Explain overlap add method for linear FIR filtering of a long sequence.** |

* Let the length of the sequence be LS and the length of the impulse response is M.
* The sequence is divided into blocks of data size having length L and M-1 zeros are appended to it to make the data size of L+M-1.
* Thus the data blocks may be represented as

x1(n) = (x(0),x(1,…..,x(L-1), 0,0,……….)

 M-1 zeros appended

x2(n)= (x(L),x(L+1),…….,x(2L-1), 0,0,……)

x3(n) = (x(2L),x(2L+1),…….x(3L-1), 0,0……..)

M-1 zeros appended

* Now L-1 zeros are added to the impulse response h (n) and N-point circular convolution is performed.
* Since each data block is terminated with M-1 zeros, the last M-1 points from each output block must be overlapped and added to the first M-1 points of the succeeding block. Hence this method is called overlap-add method.
* Let the output blocks are of the form

y1(n) = (y1(0), y1(1),……… y1(L-1,), y1(L),……………. y1(N-1))

y2(n) = (y2(0), y2(1),……… y2(L-1,), y2(L),……………. y2(N-1))

y3(n) = (y3(0), y3(1),……… y3(L-1,) , y3(L),……………. y3(N-1))

The output séquence is

y(n)=( y1(0), y1(1),……… y1(L-1,) ,y1(L)+ y2(0), y2(1),…………., y1(N-1)+ y2(M-2),y2(M),……….., y2(L)+ y3(0), y2(L+1)+ y3(1),…………………., y3(N-1))

**OVERLAP SAVE METHOD OF LINEAR FILTERING**

**Step 1 :** In this méthod L samples of the current segment and M-1 samples of the previous segment forms the input data block. Thus data block will be

X1(n) ={0,0,0,0,0,………………… ,x(0),x(1),…………….x(L-1)}

 X2(n) ={x(L-M+1), …………….x(L-1),x(L),x(L+1),,,,,,,,,,,,,x(2L-1)}

 X3(n) ={x(2L-M+1), …………….x(2L-1),x(2L),x(2L+2),,,,,,,,,,,,,x(3L-1)}

**Step2 :** Unit sample response h(n) contains M samples hence its length is made N by padding zeros. Thus

 h(n) also contains N samples.

 h(n)={ h(0), h(1), …………….h(M-1), 0,0,0,……………………(L-1 zeros)}

**Step3 :** The N point DFT of h(n) is H(k) & DFT of mth data block be xm(K) then corresponding DFT of

 output be Ym(k)

 Ym(k)= H(k) xm(K)

**Step 4 :** The sequence ym(n) Can be obtained by taking N point IDFT of Y`m(k). Initial (M-1) samples in the

 corresponding data block must be discarded. The last L samples are the correct output samples. Such

 blocks are filtered one after another to get the final output.

|  |
| --- |
| **5. Summarize the difference between overlap save and add method.** |
| **Overlap add Method** | **Overlap save Method** |
| The overlap-add procedure cuts the signal up into equal length segments with no overlap. | The overlap-save procedure cuts the signal up into equal length segments with some overlap |
| Then it zero-pads the segments and takes the DFT of the segments. Part of the convolution result corresponds to the circular convolution | Then it takes the DFT of the segments and saves the parts of the convolution that correspond to the circular convolution |
| Results in the aliasing that occurs in circular convolution. | No lost information in throwing away parts of the linear convolution. |
|  |  |
|  |  |
| Fir filtering by using the overlapping-add method | Result of circularly convolving each section with h[n]. The portions of each filter section to be discarded in forming the linear convolution are indicated |

**Challenge 9: Determine the output response y(n) if h(n)=(1,1,1), x(n)=(1,2,3,1) by using linear convolution, circular convolution & circular convolution with zero padding. (16)**

**Solution:**Linear convolution:

x (n) = {1,2,3,1}, h(n)= {1,1,1}

1 1 1 0 h(n)

1 1 1 1 0

2 2 2 2 0

3 3 3 3 0

1 1 1 1 0

 x(n)



Number of samples in linear convolution is L+M-1 = 4+3-1 = 6

**Circular convolution:**

x(n) = {1,2,3,1} ; h(n) = {1,1,1,0}

Using matrix approach



y(n) = x(n) ©h(n) ={5,4,6,6}

By comparing circular convolution output with that of linear convolution we find that the first 2 points

(M-1) are aliased ie. Last two data points are added to first two data points in linear convolution are added to first two data points as shown below. 1+4=5 & 3+1=4

**Circular convolution with zero padding:**

Add (M-1) zeros with x(n) and (L-1) zeros with h(n); x(n) = {1,2,3,1,0,0} h(n) = {1,1,1,0,0,0}



y(n) = (1,3,6,6,4,1)

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**Challenge 10: Find the output y(n) of a filter whose impulse response is h(n)=(1,1,1) and input signal x(n)=(3,-1,0,1,3,2,0,1,2,1) using over lab save and add method. [May/June-2016]**

**Solution:**

**Overlap save method:**

x1(n)= {0,0, 3,-1,0}

M-1=2 zeros L=3 data points

x2(n)= {-1,0,1,3,2}

2 data from previous 3 new data

x3(n)= {3,2,0,1,2} and x4(n)= {1,2,1,0,0}

Given h(n) = {1,1,1}

Increase the length by adding zeros( L+M-1=5)

i.e. h(n) = {1,1,1,0,0}

y1(n) = x1(n)©h(n) = {-1,0,3,2,2}



y2(n) = x2(n) © h(n) = {4,1,0,4,6}



y3(n) = x3(n) © h(n) = {6,7,5,3,3}



y4(1) = x4(n) © h(n) = {1,3,4,3,1}



[**-1,0**,3,2,2]

[**4,1,**0,4,6]

[**6,7,**5,3,3]

[**1,3,**4,3,1]

(by discarding the selected)

Y(n) = {3,2,2,0,4,6,5,3,3,4,3,1}

**Overlap add method:**

Length: L+M-1=5; N2-1=3-1=2

Therefore,

x1(n) = {3,-1,0}

x2(n) = {1,3,2}

x3(n) = {0,1,2}

x4(n) = {1,0,0}

y1(n) = x1(n).h(n) = {3,2,2,-1,0}

 3 -1 0

 1 3 -1 0

 1 3 -1 0

 1 3 -1 0

Asdy(n)={3,2,2,-1,0}

Similarly , y2(n) = x2(n) © h(n) = {1,4,6,5,2}

y3(n) = x3(n) © h(n) = {0,1,3,3,2}

y4(1) = x4(n) © h(n) = {1,1,1,0,0}

[3,2,2,-1,0]

 [1,4,6,5,2] by adding

 [0,1,3,3,2]

 [1,1,1,0,0]

y(n) = {3,2,2,0,4,6,5,3,3,4,3,1}

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

**H.W:**

**1. Using linear convolution find y(n)=x(n)\*h(n) for the sequences x(n)=(1,2,-1,2,3,-2,-3,-1,1,1,2,-1) and h(n)=(1,2).Find the result by solving overlap-save and overlap add method.**

**Solution:**

**Linear convolution:**

 **1 2 -1 2 3 -2 -3 -1 1 1 2 -1**

 1 1 2 -1 2 3 -2 -3 -1 1 1 2 -1

 2 2 4 -2 4 6 -4 -6 -2 2 2 4 -2

y(n)= (1,4,3,0,7,4,-7,-7,-1,3,4,3,-2)

2. **Perform linear convolution of finite duration sequence and by overlap-add and overlap-save methods.**

Ans: 

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**Use of FFT as Linear Filtering:**

|  |
| --- |
| **6. Write short note on use of FFT as linear Filtering.**  |

Linear filtering is needed in variety of applications. In this approach, input is known as value. If the input *x(n)* is large. Then it is very difficult to process it by using FFT algorithm. So, sectioned convolution concept is used.

LTI SYSTEM

 *x(n) y(n)=x(n)\*h(n)*

*h(n)*

The response of LTI system is given by the linear convolution of (*x(n)*input and *h(n))* impulse response. If *x(n) and h(n)* are small in length then, the computation of response is easy. But, one of the sequence *(x(n) (or) h(n))* is larger than other, then following problems may occur.

* Large amount of memory is required to store the lengthy sequence.
* Long delay occurs.

In sectioned convolution, large sequence *(x(n))* is divided into (or sectioned into) small sub sequences *((x1(n),x2(n)……..)*, Then, linear convolution of subsequences *(x1(n))*and other sequence *(h(n))* is computed. Finally, the output of all linear convolution (*(x1(n)\*h(n)),((x2(n)\*h(n)),……)* are combined to form the overall output.

**Challenge 11: Find the DFT of the sequence  for *N=4* and compute the corresponding amplitude and phase spectrum.**

**Solution:**

*Given:* The DFT of the sequence 

*Here x(0)=1, x(1)=1,x(2)=1,x(3)=0; N=4.*

*For k=0:*

**

*For k=1:*

**



*For k=2*

**



*For k=3*

**



** 

*The plot for *

 

3

 

1 0 1 2 3 *k*

0 k 

**H.W: Find the DFT of the sequence  for *N=4* and compute the corresponding amplitude and phase spectrum.**

Ans: 

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*